Grade 3 Math Unit 2-Operations and Algebraic Thinking

UNIT OVERVIEW

In Grade 3, instructional time should focus around 4 critical areas. This unit will focus on #1, Developing understanding of multiplication and division and strategies for multiplication and division within 100. (See Connections for explanation)

This unit will work within 5 clusters:
- Represent and solve problems involving multiplication and division * (See Connections for explanation)
- Understand properties of multiplaction and the relationship between multiplication and division
-Multiply and divide within 100
- Solve problems involving the four operations, and identify and explain patterns in arithmetic
- Use place value understanding and properties of operations to perform multi-digit arithmetic

STANDARDS

CC_Common Core State Standards - Mathematics (2010) - Grade 3
Domain 3.OA Operations and Algebraic Thinking
Cluster Statement: Represent and solve problems involving multiplication and division.
Standard 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.
Standard 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
Standard 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
Standard 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Cluster statement: Understand properties of multiplication and the relationship between multiplication and division.
Standard 3.OA.5 Apply properties of operations as strategies to multiply and divide.
Standard 3.OA.6 Understand division as an unknown-factor problem.

Cluster Statement: Multiply and divide within 100.
Standard 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Cluster Statement: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Standard 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Standard 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Domain 3.NBT Number and Operations in Base Ten
Cluster Statement: Use place value understanding and properties of operations to perform multi-digit arithmetic.
Standard 3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

CONTENT ELABORATIONS

3.OA.1	**3.OA.1** calls for students to interpret products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn the symbol 'x' means 'groups of' and problems such as 5 x 7 refer to 5 groups of 7. To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers and equations. Then, given an expression (e.g., 5 x 6) students interpret the expression using a multiplication context. Refer to**Table 2 in CCSS** for problem types and examples. Students should begin to use the terms, factor and product as they describe multiplication. It is important for teachers to understand there are several ways in which we can think of multiplication: 1.) Multiplication is often thought of as repeated addition of equal groups. While this definition works for some sets of numbers, it is not particularly intuitive or meaningful when we think of multiplying 3 by 1/2 or 5 by -2. In such cases, it may be helpful to widen the idea of grouping to include evaluation of part of a group. This concept is related to partitioning (which in turn, is related to division). Ex. three groups of five students can be read as 3 x 5, or 15 students, while half a group of 10 stars can be represented as 1/2 x 10, or 5 stars. These are examples of partitioning; each one of the three groups of five is part of the group of 15, and the group of 5 stars is part of the group of 10. 2.) A second concept of multiplication is that of rate or price. Ex: If a car travels four hours at 50 miles per hour, then it travels a total of 4 x 50, or 200 miles; if CDs cost eight dollars each, then three CDs will cost 3 x $8, or $24. 3.) A third concept of multiplication is that of multiplicative comparison. Ex: Sara has four CDs, Joanne has three times as many as Sara, and Sylvia has half as many as Sara. Thus Joanne has 3 x 4, or 12 CDs, and Sylvia has 1/2 x 4, or 2 CDs. **MP.1, MP.4, MP.7** should be emphasized.
3.OA.2	**3.OA.2** focuses on two distinct models of division: 1.) *Partition models* focus on the question, "How many in each group." A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag? 2.) *Measurement (repeated subtraction) models* focus on the question, "How many groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? Students need to recognize the operation of division in two different types of situations. Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor). To develop understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. See Table 2 in CCSS for problem types and examples. **MP.1, MP.4, MP.7** should be emphasized.
3.OA.3	**3.OA.3** references various strategies that can be used to solve word problems involving multiplication and division. Students should apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems. **Table 2 of CCSS** gives examples of a variety of problem solving contexts, in which students need to find the product, the group size or the number of groups. Students should be given ample experiences to explore all of the different problem structures. Students in third grade should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables). Letters are also introduced to represent unknowns in third grade. **MP.1, MP.4, MP.7** should be emphasized.
3.OA.4	This standard refers to Table 2 from CCSS for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product. The more difficult probem structures include Group Size Unknown or Number of Groups Unknown. The focus of this standard goes beyond the traditional notion of fact families, by having students explore the inverse relationship of multiplication and division. Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 x ? = 40, they think: 4 groups of some number is the same of 40, 4 times some number is the same as 40, I know that 4 groups of 10 is 40 so the unknown number is 10, the missing factor is 10 because 4 times 10 equals 40. Equations in the form of a x b=c and c=a x b should be used interchangeably, with the unknown in different positions. This standard is strongly connected to **3.OA.3** when students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation. **MP.1, MP.2, MP.6, MP.7** should be emphasized.

3.OA.5	**3.OA.5** This standard references properties of multiplication. While students do not need to use the formal terms of these properties, students should understand that properties are rules about how numbers work, students do need to be flexibly and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. The change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 x 5 x 2, a student could rearrange the numbers to first multiply 5 x 2 = 10 and then multiply 10 x 7 = 70. The commutative property (order property) states that the order of numbers does not matter when adding or multiplying numbers. For example, if a student knows that 5 x 4 = 20, then they also know that 4 x 5 = 20. Students should be introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false. - 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication) - 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1) - 3 x 6 = 6 x 3 (Commutative Property) - 2 x 3 x 5 = 6 x 5 - 10 x 2 < 5 x 2 x 2 - 2 x 3 x 5 = 10 x 3 - 1 x 6 > 3 x 0 x 2 **MP.1, MP.4, MP.7, MP.8** should be emphasized.
3.OA.6	This standard also refers to the Glossary of CCSS, Table 2 and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor in multiplication problems. Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Students use their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 32 ? = 4, students may think: - 4 groups of some number is the same as 32 - 4 times some number is the same as 32 - I know that 4 groups of 8 is 32 so the unknown number is 8 - the missing factor is 8 because 4 times 8 is 32 Equations in the form of a b = c and c = a b need to be used interchangeably, with the unknown in different positions. **MP.1, MP.7** should be emphasized.
3.OA.7	**3.OA.7** uses the word fluently, which means *accuracy, efficiency* (using a reasonable amount of steps and time) *and flexibility* (using strategies such as the distributive property). "Know from memory" does not mean focusing only on time tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the bascic facts (up to 9 x 9). By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students should have exposure to problems presented in both vertical and horizontal forms. Strategies: *multiplication by zeros and ones, Doubles* (2s facts), *Doubling twice* (4s) *Doubling three times* (8s), *Ten facts* (relating to place value, 5 x 10 is 5 tens or 50), *Five facts* (half of tens), *Skip counting, Square numbers, Nines* (10 groups less one group), *Decomposing into known facts, Turn-around facts* (Commutative Property), *Fact families and Missing factors.* **MP.2, MP.7, MP.8** should be emphasized.

3.OA.8	**3.OA.8** This standard refers to two-step problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (3.OA.7, 3.NBT.7). Adding and subtracting numbers should include numbers within 1000 and multiplying and dividing numbers should include single-digit factors and products less than 100. This standard calls for students to represent problems using equations with a letter to represent unknown quantities. It also refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50 or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness. The assessment of estimation strategies should only have one reasonable answer, or a range. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students should be expected to explain their thinking in arriving at the answer. *Estimation strategies* include, but are not limited to: - using *benchmark numbers* that are easy to compute - *front-end estimation with adjusting* (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts) - *rounding and adjusting* (students round down or round up and then adjust their estimate depending on how much the rounding changed the original values) It is important that students be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use. *Research and mathematics educators advise against providing "key words" for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through a problem and the meaning of the answer before attempting to solve it.* (M.Burns) **MP.1, MP.2, MP.4, MP.5** should be emphasized.
3.OA.9	This standard calls for students to examine arithmetic patterns involving multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increase by 2 between each term.Using a multiplication table, highlight a row of numbers and ask students what they notice about the highlighted numbers. This standard also mentions identifying patterns related to the properties of operations. Examples: - Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends - Multiples of even numbers (2, 4, 6, 8) are always even numbers - On a multiplication chart, the products in each row and column increase by the same amount (skip counting) - The multiples of any number fall on a horizontal and a vertical line due to the commutative property. **MP.1, MP.2, MP.3, MP.6, MP.7** should be emphasized.
3.NBT.3	This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects students to go beyond tricks that hinder understanding, such as "just adding zeros", and explain and reason about their products. Students use base ten blocks, diagrams, or hundreds charts to multiply one-digit numbers by multiples of 10 from 10-90. They apply their understanding of multiplication and the meaning of the multiples of 10. For example, 30 is 3 tens and 70 is 7 tens. They can interpret 2 x 40 as 2 groups of 4 tens or 8 groups of ten. They understand that 5 x 60 is 5 groups of 6 tens or 30 tens and know that 30 tens is 300. After developing this understanding they begin to recognize the patterns in multiplying by multiples of 10. **MP.2, MP.7, MP.8** should be emphasized.

UNIT VOCABULARY

equal groups	Commutative Property of Multiplication	repeated subtraction
multiplication	combination	dividend
multiplication sentence	tree diagram	divisor

multiply	division	quotient
factors	divide	inverse operations
product	partion	related facts
array	division sentence	fact family
multiple	decompose	Zero Property of Multiplication
known fact	Identity Property of Multiplication	variable
Distributive Property	expression	equation
Associative Property of Multiplication	evaluate	operations

BIG IDEAS

ENDURING UNDERSTANDINGS	ESSENTIALS QUESTIONS Choose a few questions based on the needs of your students

• Multiplication and division are inverses; they undo each other.
• Multiplication and division can be modeled with arrays.
• Multiplication is commutative, but division is not.
• There are two common situations where division may be used.
  o Partition (or fair-sharing) - given the total amount and the number of equal groups, determine how many/much in each group
  o Measurement (or repeated subtraction) - given the total amount and the amount in a group, determine how many groups of the same size can be created.
• As the divisor increases, the quotient decreases; as the divisor decreases, the quotient increases.
• There is a relationship between the divisor, the dividend, the quotient, and any remainder.
• Multiplication facts can be deduced from patterns.
• The associative property of multiplication can be used to simplify computation.
• The distributive property of multiplication allows us to find partial products and then find their sum.
• Patterns are evident when multiplying a number by ten or a multiple of ten.
• Area models are related to addition and multiplication.
• Area covers a certain amount of space using square units.
• Multiplication can be used to find the area of rectangles with whole numbers.
• Area models of rectangles and squares are directly related to the commutative property of multiplication.
● Rearranging an area such as 24 sq. units based on its dimensions or factors does NOT change the amount of area being covered (Van de Walle, pg 234). Ex. A 3 x 8 is the same area as a 4 x 6, 2 x12, and a 1 x 24.
● A product can have more than two factors.
	• What does multiplication mean?
	• What does division mean?
	• What is the importance of patterns in learning multiplication and division?
	• What strategies can be used to learn multiplication and division facts?
	• How can multiplication and division facts with smaller numbers be applied to larger numbers?

CONNECTIONS

In Critical Focus Area #1, students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

* The use of a symbol for an unknown is foundational for letter variables in Grade 4 when representing problems using equations with a letter standing for the unknown quantity **(4.OA.2, 4.OA.3)**

Standards for Mathematical Practice (SMP)
MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure (Deductive reasoning)
MP.8 Look for and express regularity in repeated reasoning (Inductive Reasoning)

REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICAITON AND DIVISION

	CONTENT	SKILLS
3.OA.1	Interpret products of whole numbers	Interpret products of whole numbers 1. find the product of multiple groups of objects 2. understand the symbol 'x' means 'groups of' 3. interpret a problem situation requiring multiplication using pictures, objects, words, numbers and equations 4. given an expression (e.g., 5 x 6) interpret the expression using a multiplication context
3.OA.2	Interpret whole number quotients of whole numbers	Interpret whole number quotients of whole numbers 1. recognize division in the partition model (How many in each group?) 2. recognize division in the measurement model (How many groups can you make?) 3. interpret a problem situation requiring multiplication using pictures, objects, words, numbers and equations 4. given a division expression (e.g., 24÷6), interpret the expression in contexts that require both interpretations of division (partition and measurement)

	CONTENT	SKILLS
3.OA.3	Use multiplication and division to solve word problems within 100	Use multiplication and division to solve word problems within 100 1. solve word problems in situations involving equal groups and arrays in which there is an Unknown Product (See CCSSM Table 2) 2. solve word problems in situations involving equal groups and arrays in which Group Size is Unknown and Number of Groups are Unknown (See CCSSM Table 2) 3. solve word problems in situations involving measurement in which there is Unknown Product, Group Size is Unknown and Number of Groups is Unknown. (See CCSSM Table 2) 4. solve one step multiplication and division word problems using multiple representations (e.g., equations, arrays, equal groups, repeated addition and subtraction, equal jumps forward or backward on a number line) 5. Represent multiplication and division problems using an equation with a symbol for the unknown number
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers	Determine the unknown whole number in a multiplication or division equation relating three whole numbers 1. Solve Unknown Product word problems that include equal groups, arrays, area and compare problems 2. Solve Group Size Unknown word problems that include equal groups, arrays, area and compare problems 3. Solve Number of Groups Unknown word problems that include equal groups, arrays, area and compare problems.

Understand properties of mulitplication and the relationship between multiplication and division

	CONTENT	SKILLS
3.OA.5	Apply properties of operations as strategies to multiply and divide.	Apply properties of operations as strategies to multiply and divide. 1. Multiply and divide within 100. 2. Explain how the properties of operations work. (Zero, Identity Property of 1, Commutative, Associative and Distributive properties) 3. Apply properties of operations as strategies to multiply and divide.
3.OA.6	Understand division as an unknown-factor problem	Understand division as an unknown-factor problem 1. Solve Group Size Unknown and Number of Groups Unknown word problems 2. Use fact family triangles to demonstrate the inverse operations of multiplication and division 3. Identify the factors on the fact family triangle and explain how the factors relate to the product and/or quotient 4. Explain the solution of a division problem by representing it as an unknown factor in a multiplication problem

MULTIPLY AND DIVIDE WITHIN 100

CONTENT	SKILLS

	CONTENT	SKILLS
3.OA.7	Fluently multiply and divide within 100	Fluently multiply and divide within 100 1. multiply within 100 accurately 2. divide within 100 accurately 3. uses a reasonable amount of steps and time to multiply 4. uses a reasonable amount of steps and time to divide 5. uses the distributive property to multiply within 100 6. uses the distributive property to divide within 100 7. uses several strategies to attain fluency of facts (pictures, manipulatives, arrays, doubles, skip counting, etc.) 8. knows from memory all products of two one-digit numbers

Solve problems involving the four operations, and identify and explain patterns in arithmetic

	CONTENT	SKILLS
3.OA.8	Solve two-step problems using the four operations.	Solve two-step problems using the four operations. 1. Know the order of operations (without parentheses). 2. Know strategies for estimating. 3. Construct an equation with a letter standing for the unknown quantity. 4. Solve two-step word problems using the four operations. 5. Justify an answer using various estimation strategies.
3.OA.9	Identify arithmetic patterns in both addition and multiplication and explain them using properties of operations	Identify arithmetic patterns in both addition and multiplication and explain them using properties of operations 1. Identify patterns in an addition table 2. Explain why patterns work in an addition table 3. Identify patterns in a multiplication table 4. Explain why patterns work in a multiplication table

Use place value understanding and properties of operations to perform multi-digit arithmetic

	CONTENT	SKILLS
3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range of 10-90	Multiply one-digit whole numbers by multiples of 10 in the range of 10-90 1. Use base ten blocks, diagrams or hundreds charts to multiply by one-digit numbers by multiples of 10 2. Apply understanding of multiplication and the meaning of multiples of 10 3. Recognize patterns in multiplying by multiples of 10

UNIT RESOURCES

1. Common Core Model Curriculum

2.Mc-Graw-Hill, My Math Chapters 4-9

3.Hands-on Standards Algebra Lessons # 5,6,7,9,10,11,& 12

4. Manipulatives: Cuisenaire Rods, 2-color counters, color tiles, centimeter cubes, number lines

5. Deb Diller Math Work Stations materials & process

6. Georgia Math Frameworks, Grade 3 Unit 2

7. **Number Talks** by Sherry Parish

8. United Streaming: Whole Number Multiplication & Division, Math Monsters:Doubles & Their Neighbors, Math Mastery: Division

9. Singapore Math

10.It's in the Cards http://illuminations.nctm.org/LessonDetail.aspx?ID=L324

11.SCS Math Resources:

Factor Game, Number Word Game (Learn your Tables),

Whack a Mole (Multiplication Skill Builders), Drag Race Division, Pony Division

12. Possible Literature: Henry and the Boy Who Thought Numbers Were Fleas by Marjorie Kaplan; Anno's Mysterious Multiplying Jar by Mitsumasa Anno